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Balancing Standstill Motorcycles by Steering Control with Feedback Delay

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Abstract:

In the development of autonomous vehicles, engineers are facing fascinating but complex tasks. The motion control of vehicles provides many challenges, even in the case of cars, but these challenges are more complicated for motorcycles. For example, balancing single-track vehicles is not straightforward at zero longitudinal speed; many exercises are also needed for humans to learn the so-called track-stand trick.

In this study, we focus on the linear stability of a riderless self-driving motorcycle, considering zero longitudinal speed. We design a linear state feedback steering controller to stabilize the motorcycle in the vertical position. For this, we use a spatial mechanical model that is based on the Whipple bicycle model (Whipple, 1899). This spatial mechanical model consists of four rigid bodies: the chassis, the fork, the front and the rear wheels, see Figure 1(a).

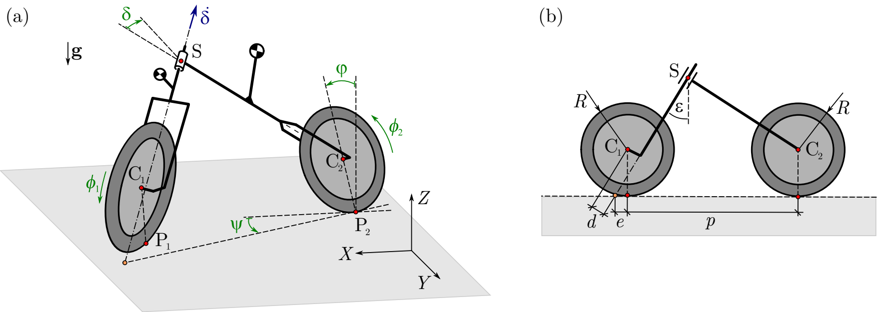

Figure 1. (a) Mechanical model of the motorcycle, (b) relevant geometric parameters in side view ($\psi = 0$, $\varphi = 0$ and $\delta = 0$).

To describe the configurational space, one has to choose seven generalized coordinates. Let us choose X and Y as the coordinates of the center point C_2 of the rear wheel, ψ as the yaw angle, φ as the lean angle, δ as the steering angle, ϕ_1 and ϕ_2 as the rotational angles of the front and the rear wheels around their own axes. In this study, we focus on the steering geometry; hence, the trail e, the rake angle ε and the fork offset d play key roles in the analysis, see Figure 1(b).

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Considering the pure rolling of the wheels, four scalar kinematic constraining equations can be formulated. Since our goal is to stabilize the motorcycle for zero longitudinal speed, the rotational speed of the rear wheel is also considered to be zero: $\dot{\phi}_2 = 0$. The governing equations are derived with Kane's method (Kane, 1985) for which we choose the lean rate and the steering rate as pseudovelocities (i.e., $\sigma_1 = \dot{\varphi}$ and $\sigma_2 = \dot{\delta}$). For zero longitudinal speed, the linearized equations of motion can be written as $\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{Q}$, where $\mathbf{M}$ is the mass matrix, $\mathbf{K}$ is the stiffness matrix, $\mathbf{x} = [\varphi \quad \delta]^{\mathrm{T}}$ and $\mathbf{Q} = [0 \quad M^{\mathrm{s}}]^{\mathrm{T}}$ with internal steering torque M^{s}. The above described governing equations agree with the literature (Meijaard, 2007). As a first step, we use a simple linear state feedback controller with feedback delay τ:

$$M^{\mathrm{s}}(t,\tau) = -P^{\mathrm{s}}_{\varphi}\varphi(t-\tau) - P^{\mathrm{s}}_{\delta}\delta(t-\tau) - D^{\mathrm{s}}_{\varphi}\dot{\varphi}(t-\tau) - D^{\mathrm{s}}_{\delta}\dot{\delta}(t-\tau)\,, \tag{1}$$

where P^{s}_{φ} and P^{s}_{δ} are proportional control gains for the lean and the steering angles, D^{s}_{φ} and D^{s}_{δ} are derivative control gains for the lean and the steering rates, respectively.

By considering zero feedback delay, i.e., $\tau = 0$, the stability boundaries can be analyzed analytically. We substitute $M^{\mathrm{s}}(t,0)$ into the equations of motion and derive the characteristic equation in the polynomial form of $b_0\lambda^4 + b_1\lambda^3 + b_2\lambda^2 + b_3\lambda + b_4 = 0$. According to the Routh-Hurwitz stability criteria, all coefficients of the characteristic equation and the third principal matrix of the Hurwitz matrix H_3 have to be positive. In Figure 2(a), the analytically obtained stability boundaries and the stable region are plotted in the plane of control gains P^{s}_{φ} and D^{s}_{φ}. The other two control gains are fixed, i.e., $P^{\mathrm{s}}_{\delta} = 100\,\mathrm{Nm}$ and $D^{\mathrm{s}}_{\delta} = 10\,\mathrm{Nms}$ and geometric parameters are based on a small-scale experimental rig (Szabo, 2021).

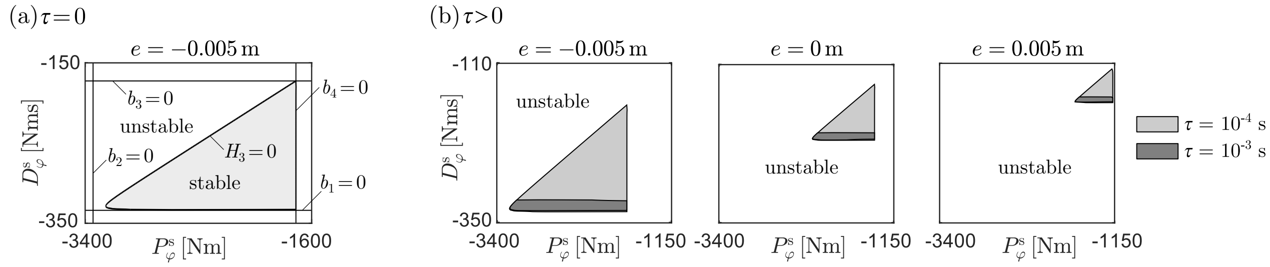

Figure 2. (a) Stability boundaries for $\tau = 0$ and $e = -0.005\,\mathrm{m}$, (b) stability charts for $\tau > 0$ and for different trail values.

When the time delay is taken into account, i.e., $\tau > 0$, the stability charts can be constructed with the help of semi-discretization (Insperger, 2011). The stable domains are shaded by gray in Figure 2(b) for $\tau > 0$ and for negative, zero and positive trail values, respectively. It is shown that the presence of a small time delay already restricts the stable domain of the control gains significantly. More importantly, having a negative trail increases the stable domain. Namely, in contrast to the requirements of the high speed stability of the motorcycle, the negative trail is beneficial for the balancing task. The experimental validation of the theoretical results is a future task, as like as the nonlinear analysis of the motorcycle balancing task.

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